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# Hydraulic Deliquoring of Compressible Filter Cakes

## Part 1: Reverse Flow in Filter Presses

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**Removal of liquid from filter cakes can be accomplished by mechanical or hydraulic methods after cake formation is complete. This paper deals with the latter procedure. The local porosity in porous beds (Tiller and Cooper, 1962) is a function of hydraulic pressure distribution and cake compressibility. Calculation of average porosity requires an integration of local values as determined by liquid flow patterns. As most compressions of filter cakes are irreversible, the local porosity is a function of the maximum effective pressure (frictional pressure loss) reached during cyclical operations. Reversal of flow through a cake develops radically changed pressure distributions which can be utilized to reduce local porosities. Analytical expressions are presented for reduction of average porosity brought about by reversing flow of liquid in plate-and-frame and recessed plate filter presses.**

## SCOPE

**Filtration engineers empirically recognize that increasing pressure results in decreases of liquid content of filter cakes. Unfortunately, highly compressible beds like colloidal silica or those arising from wastewater treatment do not respond as might be expected to direct increases in filtration pressure. Tiller and Green (1973) showed that a skin with low porosity and high flow resistance developed next to the medium during filtration of highly compressible materials. The skin deters development of frictional forces necessary to porosity reduction in a large portion of the cake. The hydraulic pressure shows little loss through much of the bed and then drops precipitously near the supporting medium. Porosity follows a similar pattern with most of the cake remaining unconsolidated while a compact layer develops where the filtrate exits.**

**To effect deliquoring of such cakes, the liquid flow can be reversed in direction at the end of filtration. Such reversal is a normal part of washing in plate-and-frame filter presses. When reverse flow takes place, the skin serves as a piston which compresses the unconsolidated portion of the cake. When liquid in the wash mode begins to flow, it encounters the resistant layer first. The large pressure drop across the skin is transmitted by interparticle forces throughout the cake and thereby results in substantial decrease in porosity.**

**Equations are developed which yield the hydraulic pressure, effective pressure, and porosity as a function of distance both for the filtration and reverse flow modes.**

## CONCLUSIONS AND SIGNIFICANCE

**Deliquoring of filter cakes is an important operation in solid-liquid separation. Mechanical squeezing utilizing membranes in filter presses requires additional investment in equipment and direct use of hydraulic pressure as developed by pumps represents the simplest method for porosity reduction. Increase in pump pressure is the preferred method if a sufficiently dry residue can be produced. Unfortunately those highly compressible cakes which cause the most difficulty in industry are little affected by elevated pressure. For the most compressible materials, increasing pressure has negligible effect on average porosity or flow rate. Consequently, methods not dependent on elevated pressures must be utilized for deliquoring such cakes.**

**Reversing the direction of flow through the highly compressible bed leads to marked reduction in porosity. In filter presses, washing automatically leads to a reverse flow pattern. However, failure to recognize the effects of hydraulic, reverse flow deliquoring can negate the practical advantage in an industrial filter. If the expressed liquid is not drained but is simply dumped along with the cake into a common receiver, the aver-**

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**age liquid context will remain unchanged.**

**Highly compressible cakes derived from wastewater are seldom washed to remove soluble substances. Consequently, presses used for such filtration must be modified to permit washing if hydraulic deliquoring is to be utilized.**

**Theoretical calculations are shown for materials with differing characteristics. Experimental verification of the theory is provided for calcium carbonate, kaolin, attapulgite, and colloidal silica. The amount of liquid which could be expressed hydraulically varies from 10–60% of the amount remaining at the end of filtration.**

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## INTRODUCTION

Deliquoring of filter cakes has become increasingly important as energy costs have mounted and environmental restrictions have become more stringent. Objectives of the deliquoring of filter cakes can be summarized as:

1. Waste materials.
   a. Make self-combustion possible in incinerators.
   b. Increase net heat obtained in energy recovery processes.
   c. Improve land fill characteristics.
   d. Reduce trucking costs.
   e. Reduce volume of hazardous compounds.
2. Useful materials.
   a. Minimize drying costs where a dry product is required.
   b. Eliminate drying when product can be used in concentrated slurry form.
   c. Reduce soluble materials.
   d. Reduce wash liquid required in subsequent operations.
   e. Lower transportation costs.

Deliquoring is accomplished by rearranging particulates in beds so that the voids are reduced by displacement of liquid in the pores. Fundamental phenomena utilized include pressure developed by mechanical or hydraulic means, interfacial forces involving the effect of surface charge on relative positioning of particles, displacement by gas under pressure, and drainage. Methods for deliquoring can be classified as:

1. Mechanical expression (Shirato et al., 1970).
   a. Membranes actuated by hydraulic pressure in filter presses (Gwilliam, 1971).
   b. Press belts on horizontal filters (Hayworth and Roberts, 1975).
   c. Pistons, screws, or rolls in units physically separated from filters or centrifuges (Bailey, 1979).
2. Hydraulic expression involving change of effective or drag pressure at points of high porosity.
   a. High filtration pressure.
   b. Reverse flow (Tiller and Risbud, 1972).
   c. Right-angled flow.
   d. Displacement with high viscosity liquids which act like a mechanical piston.
3. Combined mechanical-hydraulic. Pump pressure used to actuate membrane which covers cake surface at end of filtration (Zievers, 1980).
4. Displacement by compressed gas or steam.
5. Drainage, heating to reduce viscosity.
   a. Gravity.
   b. Vacuum.
6. Preconditioning
   a. Coagulation and flocculation.
   b. Dispersants.
7. Capillary belt to provide suction for liquid removal from cake (Lippert and Skriba, 1974).
8. Agitation and vibration to destroy particulate structures (Tiller and Cheng, 1979; Bagdasarian and Tiller, 1979).

In this paper we concentrate on hydraulic deliquoring with reverse flow.

## DELIQUORING IN FILTER PRESSES

Reverse flow deliquoring can be accomplished in plate-and-frame or recessed plate filters. A plate-and-frame filter is illustrated in Figure 1, and a recessed plate is shown in Figure 2. Discussions of the presses can be found in most unit operations textbooks and the *Chemical Engineering Handbook* (Perry and Chilton, 1973). An excellent treatment of practical problems encountered with presses was provided by Alliot (1920). In Figure 3, idealized operation of a press is shown. Slurry enters the frame and deposits cakes on plates on either side as illustrated.

Assuming that sedimentation is not an important factor, the cakes, A and B, build out from the plates and meet at the center when the press is full. The cake surfaces meet at the dotted centerline in Figure 3, and a porosity vs. distance curve is developed as shown. Porosity varies from a maximum $\epsilon_o$ at the unconsolidated cake surface to its minimum value $\epsilon_1$ at the medium where the effective pressure is a maximum and equal to the pressure drop across the cake $\Delta p_c$. In the washing mode for presses, liquid flows in one plate, passes through both cakes, and exits from the opposing plate. It traverses cake A in reverse flow with respect to the filtrate. The new effective pressure distribution developed under reverse flow conditions reduces porosity. Figure 4 illustrates how a new and similar porosity curve results. Inasmuch as the porosity is lowered, the cake volume must decrease thereby requiring that the interface at the left hand medium move to the right.

Success of the operation depends upon integral movement of the cake interface without breakup. The expressed liquid which fills

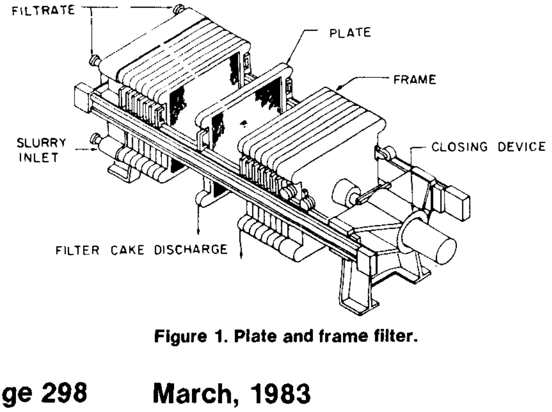

Figure 1. Plate and frame filter.

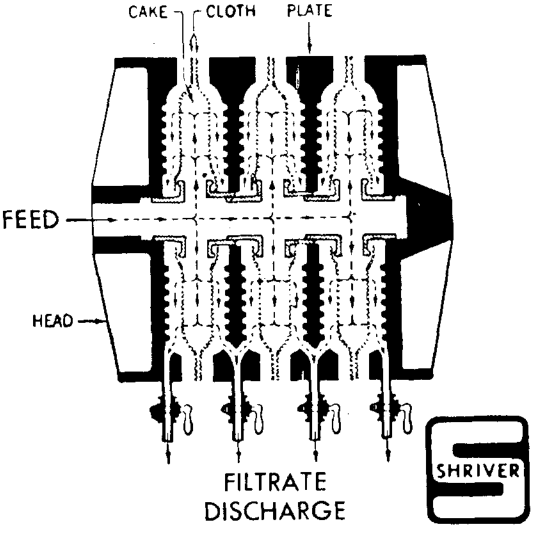

Figure 2. Recessed plate filter chamber assembly.

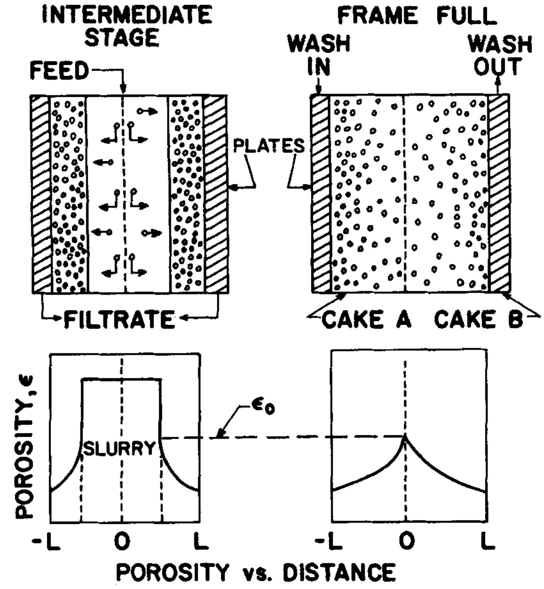

Figure 3. Cake formation in filter press.

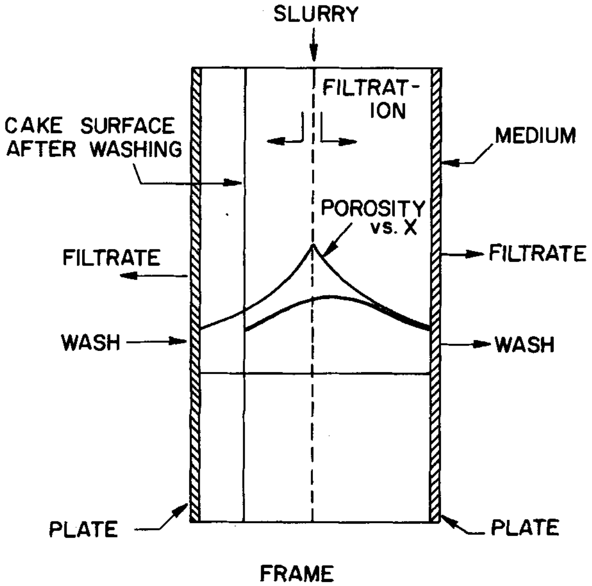

Figure 4. Deliquoring action with wash liquid.

the space between the medium and the new cake surface must be removed, or nothing will be accomplished. The liquid must be drained or blown through the cake.

We do not know how effective reverse deliquoring would be with nonuniform cakes resulting from deposition of large particles at the bottom of the frame. Washing performance is adversely affected because liquid flows preferentially through the more permeable portions of the cake. However, because deliquoring depends upon pressure difference rather than rate of flow, the effect of particle segregation would be somewhat diminished.

## CAKE STRUCTURE

Shape, size distribution, and interparticle forces contribute to the structural arrangement of particulates in filter beds. The relative positioning of particulates in porous cakes determines both porosity and permeability which are basic parameters governing behavior of solid-liquid systems such as sedimentation, filtration, centrifugation, washing, and expression. The nature of a cake is determined by a series of factors as:

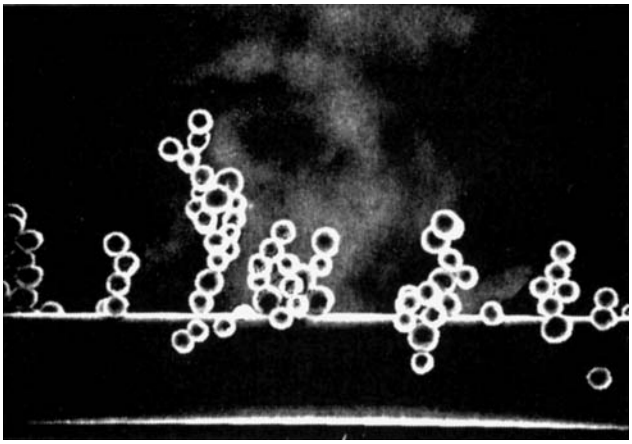

Figure 5. Initial deposition of dendrites prior to cake formation.

1. Basic particle characteristics and interaction with surrounding fluid.
2. Hydrodynamical factors affecting particle aggregation in slurry. Shear action at cake surface.
3. Initial interaction of particles with supporting medium.
4. Nature of surface structure as fresh layers are deposited.
5. Effect of drag pressure on internal cake structure as fluid flows through bed.
6. Migration of fine particles and subsequent deposition in bed or supporting medium.
7. Effect of external disturbances such as vibration or sudden pressure changes which may lead to structural rearrangement of particulates.

Interparticle forces are crucial in fixing initial particle structures. Before cake formation begins, aggregates or flocs may form. They possess their own multiparticle arrangements which mutually interact and deposit as highly porous, compressible cakes. Payatakes and Gradon (1980) have demonstrated the existence of dendritic, tree-like structures in the initial stages of cake formation as shown in Figure 5. After attachment of a particle to the receptor, streamlines are affected so that the next particle is more apt to be captured by the first particle than by the receptor itself. If interparticle attachment forces are large, sufficient resistance to shear forces results in chains of particles extending into the oncoming suspension. The dendrites ultimately interact and are compressed into cakes.

Prior dynamical treatment involving formation or breakup of particle aggregates and flow patterns at points of deposition significantly affect cake structure. Suspension concentration has an important effect upon the porosity of each surface layer as first deposited and then covered by successive waves of particles. Dilute slurries lead to more compact beds while concentrated suspensions produce open structures. With high concentrations, particulates tend to bridge and form arches whereas dilute suspensions permit individual particles to penetrate deeply into the medium or cake without interference from nearby neighbors.

Highly flocculated slurries encountered in wastewater and primary and activated sludges produce deposits which may have 90–95% voids by volume and require dewatering. When inorganic coagulants and high molecular weight organic flocculants are used, large flocs with characteristic dimensions of millimeters may settle rapidly and be removed by gravity or vacuum filtration. Loose structures (Figure 6) which retain significant quantities of liquid are frequently deliquored with press belts, recessed plates equipped with air or hydraulically actuated membranes, and high-pressure [up to 1.4 MPa (200 psi)] plate and frame or recessed plate filters.

Tiller and Green (1973) showed that highly compressible materials such as those just described produce resistant "skins" next to the medium in filter cakes. Porosity may be essentially constant over 75–80% of the cake and then drops rapidly near the medium. Under those conditions, increasing filtration pressure has little

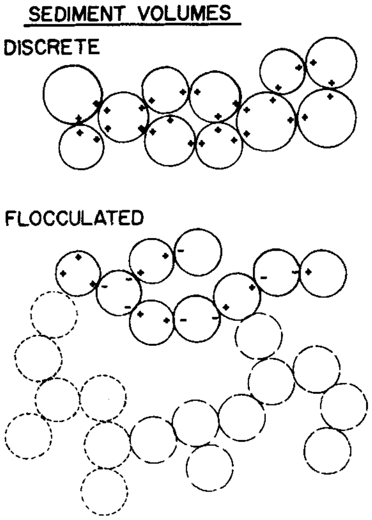

Figure 6. Sediment volumes.

effect on the unconsolidated portion of the cake. As a consequence, once some critical pressure has been reached, further increases have negligible effect on reducing average porosity or increasing flow rate. Such materials lend themselves admirably to reverse deliquoring as discussed in this paper.

When strong attractive interparticle forces or bridging mechanisms due to high molecular weight linear polymers or polymerized metal hydroxides ($Al^{+++}$ or $Fe^{+++}$) are present, loose flocs are formed [Stumm and O'Melia, 1968]. However, repulsive forces resulting from high surface charges or zeta potential prevent particles from mutual attachment. Such forces come into play at molecular distances and provide a "lubricating" effect which permits particles to glide over one another and form compact resistant cakes. A schematic representation of cakes formed with strong attractive and repulsive forces is provided in Figure 6.

## EMPIRICAL CONSTITUTIVE RELATIONSHIPS

The structure of a compressible cake is defined mathematically in terms of porosity $\epsilon$ and specific flow resistance $\alpha$ (or permeability) and their relationship to effective or compressive drag pressure $p_s$ which is simply the frictional pressure drop $p$-$p_L$ from the cake surface to an interior point in the cake (Tiller and Crump, 1977). No adequate theoretical basis is available for estimating $\epsilon$ or $\alpha$, and empirical expressions must be employed. Two sets of formulas which represent most available data involve power functions of $p_s$ (Tiller, Crump, and Ville, 1980). The first and easier set to use can be expressed as follows:

$$p_s \geq p_i$$

$$1 - \epsilon = Bp_s^\beta \qquad \alpha = ap_s^n \tag{1}$$

$$p_i \leq p_s$$

$$1 - \epsilon = 1 - \epsilon_i = Bp_i^\beta \qquad \alpha = \alpha_i = ap_i^n \tag{2}$$

The second set involves a single set of formulas for all pressures in the form:

$$1 - \epsilon = [1 - \epsilon_o][1 + p_s/p_a]^\beta \tag{3}$$

$$\alpha = \alpha_o[1 + p_s/p_a]^n \tag{4}$$

where $p_a$ is an empirical constant with dimensions of pressure. Shirato et al. (1970) used Eq. 3 and Carman (1938) suggested Eq. 4 as an alternative to Eq. 1. There are insufficient data of sufficient reliability to justify use of one set of equations as opposed to the other. Tiller and Leu (1980) presented graphical procedures for finding simultaneously the three independent parameters for each set of formulas.

Equations 1 and 2 fit traditional expressions found in the literature but suffer from requiring different formulations above and below an empirically chosen $p_i$. They also assume that porosity and specific flow resistance are constant in the low pressure range where, in fact, the greatest rate of change occurs. Equations 3 and 4 provide a smooth transition over the entire pressure range but are less familiar to practioners in the field of solid-liquid separation. The latter group is used in this paper.

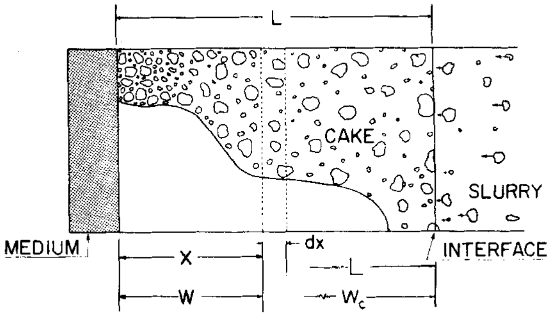

Figure 7. Schematic cake.

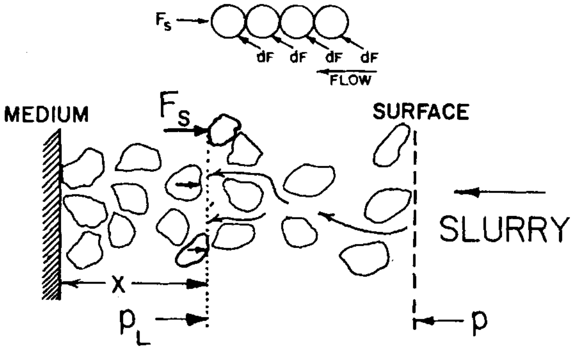

Figure 8. Frictional drag in cakes.

The parameters $\alpha_o$ and $\epsilon_o$ are specific flow resistance and porosity in an unstressed bed. They represent values of $\alpha$ and $\epsilon$ in layers as they are deposited at the cake surface and before they are subjected to frictional drag resulting from flow through the cake. They are determined by particle characteristics and fluid-particle dynamics at the cake surface. The powers $n$ and $\beta$ are compressibility coefficients. Little is known about their relationship to properties of particulates. As a rough approximation in examples worked in this paper, it will be assumed that $n = 4\beta$.

## FRICTIONAL DRAG ON PARTICLES

In reverse deliquoring, each particle in the cake is subjected to skin and form drag. Frictional surface forces generate internal forces which are communicated from particle to particle at points of contact. An ideal particulate model consists of particles which have point as opposed to area contact. Where area contact exists, the equivalent of tube walls pass through the porous bed as exemplified by porous metals and ceramic filters. In the ideal particulate bed, the liquid pressure is assumed to be effective over the entire surface. As the liquid flows around a particle, the normal pressure and tangential drag components can be integrated over the entire surface. Porosity decrease is caused by the accumulated frictional and form drags.

A force balance over the distance $L$-$x$ (Figures 7 and 8) assuming negligible momentum changes can be written as

$$Ap_L + F_s = Ap \tag{5}$$

where it is assumed that the liquid pressure $p_L$ is active over the entire cross-section of the cake, and $p$ is the filtration pressure.

Equation 5 rests on the assumptions that there is point contact among particles and that the liquid pressure integrated over an undulating unbroken surface yields the first term in Eq. 5. The term $F_s$ represents the accumulated frictional drag arising from the total particulate surface. It is communicated from particle to particle at points of contact. A fictitious pressure (effective or compressive drag pressure) is defined by $p_s = F_s/A$ and may be calculated from the expression

$$p_s = p - p_L \tag{6}$$

If we take differentials with respect to $x$ at constant time in the interior of the cake, we have:

$$dp_s + dp_L = 0 \tag{7}$$

Equation 7 means that drag pressure increases and hydraulic pressure decreases as we move from the upstream face of the cake toward the other side of the filter medium.

In general, the applied pressure is a function of $t$, and $p_s$ and $p_L$ are functions of $x$ and $t$. It is a fundamental assumption of flow through compressible, porous media that porosity and specific flow resistance are unique functions of $p_s$ and those conditions which fix the initial cake structure $[\alpha_o, \epsilon_o]$ at $p_s = 0$. It is further postulated that compression is irreversible and that porosity is determined by the maximum value of $p_s$ reached at any given time.

This last assumption underlies the theoretical basis of reverse deliquoring.

## POROSITY DISTRIBUTION DURING FILTRATION

Assuming one dimensional flow and neglecting movement of the solids, Darcy's law can be written in spatial coordinates in the form

$$\frac{dp_L}{dx} = -\frac{dp_s}{dx} = \frac{\mu q}{K} \tag{8}$$

where $dp_s/dx$ is the effective pressure gradient, $q$ is the superficial flow rate/area, and $K$ is the permeability. We assume that the solids are not moving in Eq. 8; otherwise a relative velocity term (Shirato et al., 1969) would have to be introduced. Material coordinates can be introduced by means of

$$dw = \rho_s[1 - \epsilon]\, dx \tag{9}$$

where $w$ represents the mass of dry solids per unit area. Eliminating $dx$ from Eq. 8 yields

$$\frac{dp_s}{dw} = -\frac{\mu q}{\rho_s[1 - \epsilon]K} = -\mu\alpha q \tag{10}$$

which serves to define the local specific flow resistance $\alpha$. Combining these equations and integrating $dx$ yields the following:

$$\int_0^x dx = x = \frac{1}{\mu q \rho_s} \int_{p_s}^{\Delta p_c} \frac{dp_s}{\alpha(1 - \epsilon)} \tag{11}$$

Integrating over the entire cake and dividing the result into Eq. 11 produces (Tiller and Cooper, 1962)

$$\frac{x}{L} = 1 - \frac{\displaystyle\int_0^{p_s} dp_s/\alpha(1 - \epsilon)}{\displaystyle\int_0^{\Delta p_c} dp_s/\alpha(1 - \epsilon)} \tag{12}$$

This equation provides a general relationship between $p_s$ and the fractional distance through the cake. Substituting Eqs. 3 and 4 into Eq. 12 and integrating leads to (Tiller, Crump and Ville, 1980):

$$\frac{x}{L} = 1 - \frac{(1 + \pi_s)^{1-n-\beta} - 1}{(1 + \Delta\pi_c)^{1-n-\beta} - 1} \tag{13}$$

where $\pi_s = p_s/p_a$ and $\Delta\pi_c = \Delta p_c/p_a$. In this paper, it will be assumed that $\Delta p_c = p$. The dimensionless effective pressure can be eliminated from Eq. 13 through use of Eq. 3 to give a direct relationship between porosity and distance. Equation 3 was used to calculate $\epsilon$ and $p_s$ as functions of $x$.

The average porosity may be calculated with the use of the following equations (Shirato and Okamura, 1955; Tiller and Cooper, 1962).

$$\epsilon_{av} = 1 - \int_0^{\Delta p_c} \frac{dp_s}{\alpha} \Big/ \int_0^{\Delta p_c} \frac{dp_s}{\alpha(1 - \epsilon)} \tag{14}$$

Substituting Eqs. 3 and 4 in Eq. 14 and integrating leads to

$$\epsilon_{av} = 1 - \frac{(1 - n - \beta)(1 - \epsilon_o)[(1 + \Delta\pi_c)^{1-n} - 1]}{(1 - n)[(1 + \Delta\pi_c)^{1-n-\beta} - 1]} \qquad n \neq 1 \tag{15}$$

$$= 1 - \frac{\beta(1 - \epsilon_o)\, ln\,(1 + \Delta\pi_c)}{1 - (1 + \Delta\pi_c)^{-\beta}} \qquad n = 1 \tag{16}$$

The average filtration resistance, $\alpha_{av}$, is defined by

$$\frac{1}{\alpha_{av}} = \frac{1}{\Delta p_c} \int_0^{\Delta p_c} \frac{dp_s}{\alpha} \tag{17}$$

With thick cakes, the resistance of the medium is generally small compared to the cake; and the pressure drop across the cake $\Delta p_c$ can be assumed equal to $p$. Letting $\Delta p_c = p$ yields

$$\frac{1}{\alpha_{av}} = \frac{p_a}{(1 - n)\alpha_o p} \left[\left(1 + \frac{p}{p_a}\right)^{1-n} - 1\right] \qquad n \neq 1 \tag{17a}$$

$$\frac{1}{\alpha_{av}} = \frac{p_a}{\alpha_o p}\, ln\,(1 + p/p_a) \qquad n = 1 \tag{17b}$$

Equation 17 does not represent a correct mathematical definition of an average, since $(1/y_{av}) \neq (1/y)_{av}$ in general, and $(1/\alpha_{av})$ should be replaced by $(1/\alpha)_{av}$. However, this definition is widely accepted, and we shall not attempt to change current practice.

## CONSTANT-PRESSURE REVERSE DELIQUORING

In Figure 9 conditions for filtration and reversed flow cycles are illustrated. The dotted curves marked "old $p_s$" and "old $p_L$" show values at the end of filtration. In the bottom half of Figure 9, the "old $\epsilon$" curve represents the porosity variation at the time the press is filled. During deliquoring, the cake interface ultimately assumes a new position indicated by "new cake surface." The new liquid and effective pressures and porosity distributions are drawn as solid curves.

We shall consider the effect of reversing the flow in a wash mode under different pressures, $p_2$, less than, equal to, and greater than the final filtration pressure, $p$. Only the final equilibrium porosity and pressure curves will be calculated. Figure 9 reflects the special case in which the deliquoring pressure $p_2$ is less than $p$.

As liquid in reverse flow enters at the cake-medium interface, a bed structure is encountered which was established during the

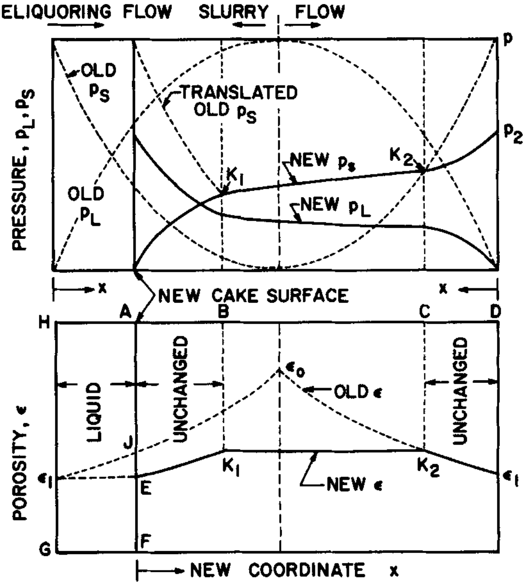

**Figure 9. Porosity and local pressures during filtration and deliquoring.**

filtration cycle. It is assumed that the compressive forces act irreversibly, and as long as $p_s$ of reverse deliquoring is less than its value during filtration at a given point, the porosity retains its original value. The compressive drag pressure rises throughout the bed in accord with the drop of hydraulic pressure.

The effective pressure rises without changing the cake structure until it reaches $K_1$ where the new $p_s$ equals the old $p_s$. The unchanged portion of the cake, *AB*, moves to the right and assumes its new position in which the "translated old $p_s$" governs local values of $\epsilon$ and $\alpha$. After $K_1$ is passed and $p_s > p_s(K_1)$, the cake takes on lower values of porosity as illustrated by the "new $\epsilon$" curve lying between $K_1$ and $K_2$. At $K_2$, the new $p_s$ curve intersects the old a second time, and another region is entered in which the structure is maintained in its original filtration state. Thus, the cake only changes in the region *BC* and retains its original porosity in *AB* and *CD*. If the reverse deliquoring pressure had been chosen so that $p_2 > p$, the intersection $K_2$ would not exist; and there would be no undisturbed cake in the region *CD*.

When $p_2 < p$, the value of the porosity at the medium $\epsilon_1$ remains unchanged at both interfaces. If $p_2 > p$, the porosity $\epsilon$, on the righthand cake surface would decrease in accord with the new pressure drop. The porosity curves $\epsilon_1 J$ and $EK_1$ are identical. A volumetric balance requires that the expressed liquid represented by *AJEFGH* equal that enclosed by $\epsilon_1 J \epsilon_0 K_2 K_1 E \epsilon_1$.

## DELIQUORING FORMULAS

Mathematical solution of the reverse deliquoring problem requires numerical solutions in which the cake is divided into three regions for cases in which $p_2 < p$ and two regions for $p_2 \geq p$.

*Case 1:* Deliquoring pressure greater than filtration pressure, $p_2 \geq p$.

Referring to Figure 9, we note that the cake in distance AB is unchanged during deliquoring and its average resistance will be the same as during filtration. As $p_2 > p$, the section *CD* will be missing; and *C* and *D* will coincide. Cake in the region *CD* will be unchanged and is important when $p_2 < p$. In the region *BC*, the new $p_s$ exceeds the old values; and the porosity is decreased. We shall obtain an expression which yields the ratio of the final rate of filtration $q$, to the final rate of deliquoring $q_2$. Based upon the formulas which will be developed, $p_s(K_1)$ can be predicted. The entire porosity distribution then follows.

During filtration the rate of flow/area $q_1$ can be related to the cake in *AB* which remains unchanged during deliquoring by

$$\mu q_1 = \frac{1}{w_{AB}} \int_{p_s(K_1)}^{p} dp_s/\alpha = \frac{p - p_s(K_1)}{w_{AB}\alpha_{AB}} \tag{18}$$

During deliquoring after steady-state has been reached, the resistance of *AB* is unchanged and $q_2$ is given by

$$\mu q_2 = \frac{1}{w_{AB}} \frac{p_s(K_1)}{\alpha_{AB}} = \frac{1}{w_{AB}} \frac{p_s(K_1)}{p - p_s(K_1)} \int_{p_s(K_1)}^{p} dp_s/\alpha \tag{19}$$

The ratio becomes

$$\frac{q_2}{q_1} = \frac{p_s(K_1)}{p - p_s(K_1)} \tag{20}$$

Another expression for this ratio can be obtained by considering the entire cake. During filtration

$$\int_0^{2w_c} \mu q_2 dw = \frac{p_s(K_1)}{p - p_s(K_1)} \int_{p_s(K_1)}^{p} dp_s/\alpha + \int_{p_s(K_1)}^{p_2} dp_s/\alpha \tag{21}$$

The limits on the last integral are changed to give two integrals with limits $[p_s(K_1), p]$ and $(p, p_2)$. Then dividing the resulting equation by Eq. 21 yields

$$\frac{2q_2}{q_1} \int_0^{p} \frac{dp_s}{\alpha} = \frac{p_s(K_1)}{p - p_s(K_1)} \int_{p_s(K_1)}^{p} \frac{dp_s}{\alpha} + \int_{p}^{p_2} \frac{dp_s}{\alpha} \tag{22}$$

Substituting for $q_2/q$, and rearranging produces

$$\frac{1}{p}\left[\int_0^{p_2} \frac{dp_s}{\alpha} + \int_0^{p} \frac{dp_s}{\alpha}\right] = \frac{1}{p_s(K_1)} \int_{p_s(K_1)}^{p_2} dp_s/\alpha \tag{23}$$

Substituting Eq. 4 into Eq. 23 and integrating leads to ($n \neq 1$)

$$\frac{1}{p}\left[\left(1 + \frac{p_s}{p_a}\right)^{1-n} + \left(1 + \frac{p}{p_a}\right)^{1-n} - 2\right] = f(p, p_2)$$
$$= \frac{1}{p_s(K_1)}\left[\left(1 + \frac{p_2}{p_a}\right)^{1-n} - \left(1 + \frac{p_s(K_1)}{p_a}\right)^{1-n}\right] \tag{24}$$

Values of $p_s(K_1)$ are obtained from this equation or from Eq. 24 if $\alpha$ vs. $p_s$ relations different from Eq. 4 are used.

We next obtain the amount of liquid squeezed from the cake. The new cake thickness is $L_2 = x_{AB} + x_{BC}$ where $x_{AB}$ is the undisturbed cake thickness; using Eq. 11, we obtain

$$L_2 = \frac{1}{\mu q_1 \rho_s} \int_{p_s(K_1)}^{p} dp_s/\alpha(1 - \epsilon) + \frac{1}{\mu q_2 \rho_s} \int_{p_s(K_1)}^{p_s} dp_s/\alpha(1 - \epsilon) \tag{25}$$

Eliminating $q_1$ and $q_2$ leads to

$$\frac{L_2}{L} = \frac{p \displaystyle\int_{p_s(K_1)}^{p_2} \frac{dp_s}{\alpha(1 - \epsilon)} - p_s(K_1) \int_{p}^{p_2} \frac{dp_s}{\alpha(1 - \epsilon)}}{p_s(K_1) \displaystyle\int_0^{p} dp_s/\alpha(1 - \epsilon)} \tag{26}$$

Substituting Eqs. 3 and 4 into Eq. 26 and integrating yields

$$\frac{L_2}{L} = \frac{p[\psi_2^{1-n-\beta} - \psi_k^{1-n-\beta}] - p_s(K_1)[\psi_2^{1-n-\beta} - \psi^{1-n-\beta}]}{p_s(K_1)[\psi^{1-n-\beta} - 1]} \tag{27}$$

where $\psi_2 = (1 + p_2/p_a)$, $\psi_k = (1 + p_s(K_1)/p_a)$, and $\psi = (1 + p/p_a)$. The fraction of liquid squeezed out of the cake is

$$F = 1 - L_2\epsilon_{2av}/2L\epsilon_{av} \tag{28}$$

where $\epsilon_{av}$ and $\epsilon_{2av}$ are the average porosities respectively at the end of filtration and deliquoring. The value of $\epsilon_{av}$ can be calculated with Eqs. 15 and 16. For the deliquored cake, Eq. 14 must be applied separately to each section of the cake.

*Case 2:* Deliquoring pressure less than filtration pressure, $p_2 < p$.

A procedure similar to the one developed for $p_2 > p$ is followed with the addition of the undisturbed portion *CD* and the second unknown effective pressure $p_s(K_2)$. The first set of equations involving the ratio $q_2/q_1$ takes the form

$$\frac{q_2}{q_1} = \frac{p_s(K_1)}{p - p_s(K_1)} = \frac{p_2 - p_s(K_2)}{p - p_s(K_2)} \tag{29}$$

The equation analogous to Eq. 22 is

$$\frac{q_2}{q_1} = \frac{\displaystyle\int_{p_s(K_1)}^{p_s(K_2)} \frac{dp_s}{\alpha}}{\displaystyle\int_0^{p_s(K_1)} \frac{dp_s}{\alpha} + \int_0^{p_s(K_2)} \frac{dp_s}{\alpha}} \tag{30}$$

Both $p_s(K_1)$ and $p_s(K_2)$ must be obtained by solving Eqs. 29 and 30 simultaneously. Details of the equations for Case 2 were developed by Horng (1981).

## CALCULATED VALUES

Calculations have been made for various values of the resistance compressibility factor $n$ and the permeability compression factor $\beta$ by assuming $n = 4\beta$ or $n + \beta = 1.25\,n$. This relation is based upon data for a half dozen substances and should not be given undue importance. There are very few reliable values for $n$ and $\beta$ in the literature and no correlations with microscopic properties of particulates. Values of $n$ = 0.2, 2/3, 1, and 1.5 will be used for illustrative purposes. Reasonable values of $\epsilon_o$ have been chosen with recognition given to the fact that high values of $n$ are usually accompanied by large porosities. Filter aids such as diatomaceous earth and expanded perlite are notable exceptions. Although their porosities are in the range of 0.85–0.9, the resistance compressibility

TABLE 1. COMPRESSIBILITY CHARACTERISTICS

| $n$ | $\epsilon_o$ | Description | Characteristics |
|---|---|---|---|
| 0.2 | 0.55 | Slightly Compressible | Increasing pressure has large effect on rate and little effect on cake porosity. Typical of filter aids and cakes with large particles except that $\epsilon_o$ is in the range of 0.8–0.9 for filter aids. |
| 2/3 | 0.75 | Normally Compressible | Pressure has moderate effect on rate and porosity. Typical of clays. |
| 1.0 | 0.85 | Very Compressible | Pressure has little effect on rate. |
| 1.5 | 0.90 | Extremely Compressible | A hard skin is formed close to the medium and 80–90% of the cake is unconsolidated. Beyond some usually low value, pressure has little effect on either rate or average cake porosity (Tiller and Green, 1973). Typical of flocculated latex, colloidal silica, and waste sludges. |

coefficients are apt to lie near to 0.2.

In terms of cake characteristics, a qualitative relationship of compressibility coefficients to normally used terms is provided in Table 1.

Calculations have been made in terms of dimensionless variables. The filtration pressure was chosen such that $\Delta\pi_c = \Delta p_c/p_a = p/p_a = \pi = 50$. For reverse deliquoring values of $p_2/p_a = 5$, 50 and 100 were utilized for the graphs.

In Figure 10, the effective or compressive solid pressure is plotted against fractional distance through the frame for $n = 0.2$ and 1.0. The curves marked zero represent the original $p_s$ distribution at the end of filtration. Points marked $A$ correspond to the position of the cake surface after deliquoring. The distance from $A$ to the righthand edge of the graph represents the deliquored cake thickness. The points $A_{50}$ lie at the position of the interface after deliquoring is completed. The subscript 50 in $A$ refers to the deliquoring pressure. It is clear that there is little movement of the cake when $n = 0.2$ whereas there is substantial deliquoring for $n = 1.0$ as indicated by the interface position at $x/L_1 = -0.45$.

The distances $A_5B_5$ and $C_5$ to $x/L = 1$ represent the portions of the cake which are unchanged from filtration Points $D_{50}$ and $D_{100}$ corresponding to $K_1$ on Figure 9. Both $D_{50}$ and $D_{100}$ are translated to $B_{50}$ and $B_{100}$ during deliquoring. From $A_{50}$ to $B_{50}$, we see the effective pressure use in the unchanged portion of the cake containing mass/area $w_{AB}$.

Plots of porosity vs. $x/L$ are shown in Figure 11. For the highly compressible material with $n = 1.5$, the $\epsilon$ vs. $x/L$ curve is convex at the end of filtration where the curve is marked zero. With the reverse dimensionless deliquoring pressure $\pi_2 = 5$, there are three distinct portions of the curves shown as $AB$, $BC$ and $CD$. The regions identified by $AB$ and $CD$ represent unchanged parts of the cake. All of the deliquoring occurs in $BC$. Impressive quantities of liquid are removed as can be seen by the position of the interface.

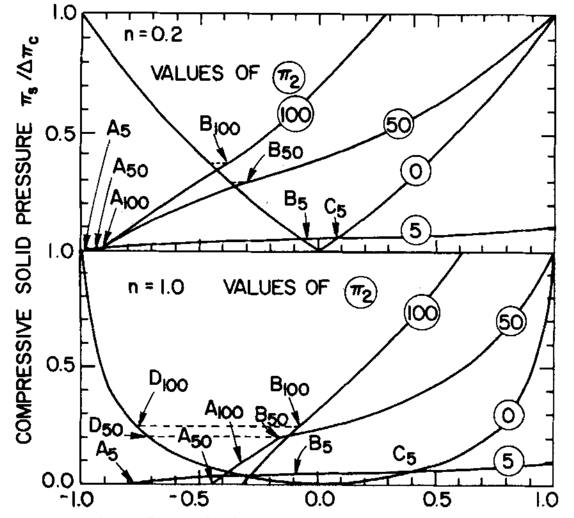

**Figure 10. Compressive drag or effective pressure during filtration and deliquoring.**

The rapid porosity decrease near the media for $n = 1.5$ is indicative of skin formation which is adverse to development of high flow rates during filtration with increasing pressure. Average porosity is also little affected after a critical (usually low) pressure drop is reached across such cakes. Whereas increasing pressure in highly compressible cakes is ineffective above a certain pressure because of the resistant skin, the opposite is true for reverse deliquoring. The skin serves as a relatively impermeable membrane and effectively compresses the rest of the cake.

In Figure 12, the fraction of liquid squeezed from the cakes is shown as a function of the compressibility $n$ and the reverse deliquoring pressure. In normal filtration practice, one might expect to remove from 10–50% of the liquid in the cake with reverse deliquoring.

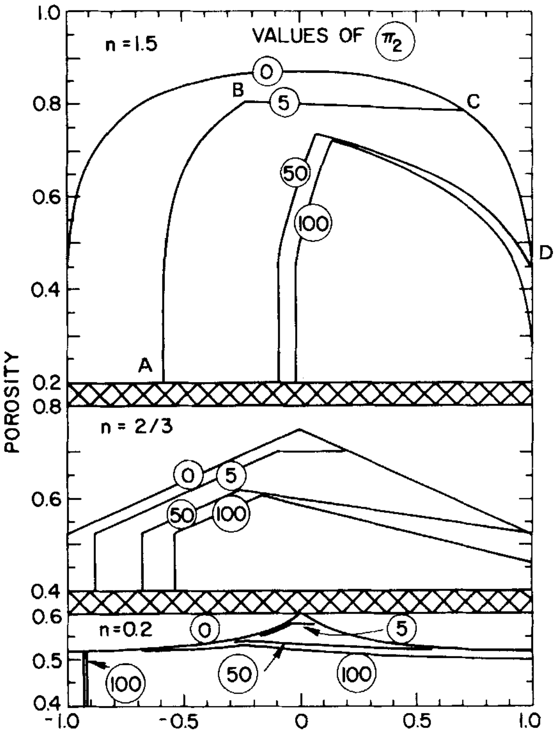

**Figure 11. Porosity variation during deliquoring.**

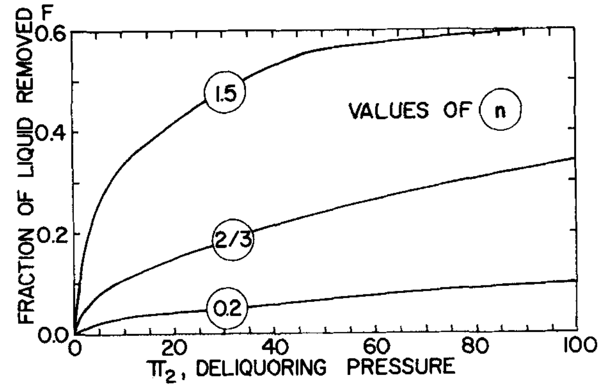

**Figure 12. Theoretical fraction of liquor removed as a function of compressibility.**

### TIME FOR DELIQUORING

Precise calculation of deliquoring time would require solution of partial differential equations. Inasmuch as the reverse flow rate decreases with time, an upper limit can be obtained by dividing the volume of expressed liquid by the final flow rate calculated for steady-state flow through the deliquored cake.

The ratio of the final deliquoring rate $q_2$ to the final filtration rate $q_1$ can be obtained from Eq. 22 or 30. We assume

$$q_2 = \xi q_1 \tag{31}$$

where usually $\xi < 1$. The time required for expression would be

$$t_2 = 2F\epsilon_{av}L/\xi q_1 \tag{32}$$

As the medium resistance has been assumed to be negligible in our calculations, filtrate volume is related parabolically to time by

$$v^2 = 2pt_1/\mu c\alpha_{av} \tag{33}$$

where $c = \rho_s/(1 - s/s_c)$ comes from a material balance over slurry, filtrate, and cake (Tiller and Crump, 1977). The quantity $c$ equals the mass of dry solids deposited in the cake per unit volume of filtrate. The thickness in Eq. 32 can be related to $v$ in Eq. 33 by

$$w_c = cv = \rho_s(1 - \epsilon_{av})L \tag{34}$$

We now substitute $q_1 = dv/dt$ from Eq. 33 and $L$ from Eq. 34 into Eq. 32 to obtain

$$t_2 = \frac{4F\epsilon_{av}c}{\xi\rho_s(1 - \epsilon_{av})}t_1 \tag{35}$$

Substituting for $\epsilon_{av}$ in terms of $s_c$ in accord with

$$\epsilon_{av} = \frac{(1 - s_c)/\rho}{(1 - s_c)/\rho + s_c/\rho_s} \tag{36}$$

leads to

$$t_2 = \frac{4F}{\xi}\frac{s}{s_c}\frac{1 - s_c}{1 - s/s_c}t_1 \tag{37}$$

Typical values substituted in Eq. 37 yield values of 0.1–0.2 for $t_2/t_1$ which provides a rough idea of the time penalty attached to reverse deliquoring.

### EXPERIMENTAL RESULTS

Two types of experiments were necessary to verify the theoretical equations. In the first set, rates were determined at a series of pressures ranging from 50 to 100 kPa for thicknesses up to 35 mm. Basic parameters calculated for four materials are listed in Table 2.

TABLE 2. FILTRATION PARAMETERS

| Materials | $n$ | $\alpha_o$(m/kg) | $p_a$(kPa) | $1 - \epsilon_o$ | $\beta$ |
|---|---|---|---|---|---|
| $CaCO_3$ | 0.25 | 1.5($10^{11}$) | 10 | 0.2 | 0.08 |
| Kaolin | 0.55 | 1.1($10^{11}$) | 19 | 0.32 | 0.09 |
| Attapulgite | 1.3 | 6.0($10^{11}$) | 17 | 0.077 | 0.4 |
| Colloidal Silica | 1.5 | 1.0($10^{12}$) | 17 | 0.07 | 0.4 |

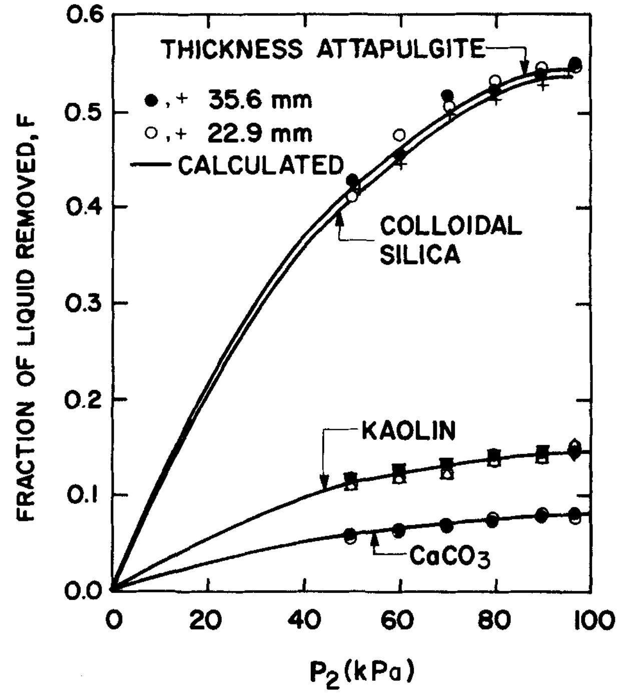

**Figure 13. Fraction of liquor removed in vacuum filtration and deliquoring.**

In the second series of experiments, two horizontal cakes were prepared simultaneously. One cake was inverted and placed upon top of the other to simulate cake formation in a filter press. The liquid was run through the cakes in series under varying pressures.

Cakes were prepared with thicknesses of 22.4 and 35.6 mm at a pressure of 81.3 kPa. Deliquoring pressures ranged from 50–100 kPa. On a normalized thickness basis, experiments with the 22.4 and 35.6 mm cakes gave nearly identical results.

In Figure 13, the fraction of liquid removed is plotted against the reverse deliquoring pressure along with the theoretical values calculated with the equations previously developed.

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### NOTATION

$a$ = empirical constant in Eq. 1, dimensions meaningless
$A$ = cross-sectional area, m$^2$
$B$ = empirical constant in Eq. 2, dimensions meaningless
$c$ = mass of solids/volume of filtrate, kg/m$^3$
$F$ = fraction of liquid removed from cake, dimensionless
$F_s$ = accumulative drag force on particles from 0 to $x$, N
$K$ = permeability, m$^2$
$L$ = cake thickness during filtration, m
$L_2$ = total cake thickness of all solids in frame (2 cakes) after deliquoring, m

$n$ = empirical constant in Eq. 1, dimensions meaningless, or in Eq. 4, dimensionless
$p$ = applied pressure during filtration, Pa
$p_i$ = pressure below which $\alpha$ and $\epsilon$ are assumed constant, Pa
$p_a$ = empirical constant in Eqs. 3 and 4, Pa
$p_L$ = hydraulic pressure at $x$, Pa
$p_s$ = solids compressive or effective pressure at $x$, Pa
$p_s(K_1)$ = solids compressive pressure at $K_1$, Pa
$p_1$ = pressure required to overcome medium resistance, Pa
$p_2$ = pressure during deliquoring, Pa
$\Delta p_c$ = pressure drop across cake, Pa
$q$ = superficial flow rate/area, $m^3/m^2 \cdot s$
$q_1$ = final flow rate at end of filtration, $m^3/m^2 \cdot s$
$q_2$ = final flow rate at end of deliquoring, $m^3/m^2 \cdot s$
$x$ = distance from origin, m
$x_{AB}$ = thickness of portion of cake unchanged during deliquoring, m
$x_{BC}$ = thickness of portion of cake compressed during deliquoring
$w$ = mass of dry cake/area, $kg/m^2$
$w_{AB}$ = mass of dry cake/area in $x_{AB}$, m
$w_c$ = total mass of dry cake/area, m

### Greek Letters

$\alpha$ = local specific flow resistance, m/kg
$\alpha_i$ = empirical constant, m/kg
$\alpha_o$ = value of $\alpha$ at $p_s = 0$, m/kg
$\alpha_{av}$ = average value of $\alpha$ defined by Eq. 17, m/kg
$\alpha_{AB}$ = average value of $\alpha$ of cake in distance $x_{AB}$, m/kg
$\beta$ = empirical constant in Eq. 2, dimensionless
$\epsilon$ = local porosity, dimensionless
$\epsilon_i$ = empirical constant in Eq. 2, dimensionless
$\epsilon_o$ = value of $\epsilon$ at $p_s = 0$, dimensionless
$\epsilon_{av}$ = average value of $\epsilon$ at end of filtration, dimensionless
$\epsilon_{2av}$ = average value of $\epsilon$ at end of deliquoring, dimensionless
$\mu$ = viscosity, Pa·s
$\xi$ = ratio of $q_2/q_1$, dimensionless
$\pi_s$ = dimensionless effective pressure, $p_s/p_a$
$\rho_s$ = true density of solids, $kg/m^3$
$\Delta\pi_c$ = dimensionless pressure drop across cake, $\Delta p_c/p_a$
$\psi_2$ = value of $(1 + p_2/p_a)$, dimensionless
$\psi_k$ = value of $\psi$ at point $K$, dimensionless

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